University of EL-Oued
Faculty of exact sciences
Computer science department
2nd year License LMD –Computer science

Session: January 2026
Module: Mathematical Logic
Duration: 1h30
Date: 18/01/2026 à 12h30.
02 Pages

Exam N° : 01

No documentation allowed

Exercise 1: True/False Questions

For each statement, answer **True (T)** or **False (F)**. Correct the statement if it is false.

1. In propositional logic, every consistent formula is also a tautology.
2. In propositional logic, if $A \models B$ then $A \rightarrow \neg B$ is a tautology.
3. Using resolution with refutation to prove $\Gamma \models C$, we try to derive the empty clause from $\Gamma \cup \{C\}$ after conversion to clausal form.
4. Forward chaining is not a resolution method.
5. If a set of clauses is consistent, then the empty clause can be derived using resolution.
6. Skolemization is used to find the clausal form in propositional logic.

Exercise 2: Propositional Logic

Consider the following knowledge base Γ:

1. $P \Rightarrow (T \wedge R)$
2. $(S \vee T) \Rightarrow P$
3. $T \Leftrightarrow U$
4. S
5. $\neg U$

Question 2.1: Transform Γ into **Horn clause.**

Question 2.2: Proving U from Γ (only the horn clauses) using **backward chaining method**, Show the steps clearly.

Exercise 3: Predicate Logic

Let the following be given:

- C(x): x is a student in computer science
- L(x,y): x likes module y
- R(x, y) : x revises the exam y
- P(x,y): x will pass the exam y
- a: Ahmed
- ml: mathematical logic

Consider F:

1. $\forall x(C(x) \Rightarrow \exists y L(x,y))$
2. $\exists y \forall x (L(x,y) \vee R(x,y) \Rightarrow P(x,y))$
3. $\forall x \forall y R(x,y)$
4. $\exists x C(x)$

Question 3.1: Translate each formula into naturel language (text).

Question 3.2: Translate the statement C: **"Ahmed passed the exam of mathematical logic"** into first-order predicate logic.

Question 3.3: Convert F into clausal form, show each step.

Question 3.4: Using the **refutation method**, prove that: $F \models C$, Show the steps of resolution clearly.

Good Luck

Exam Solutions & Marking Scheme (/20 points)

Exercise 1 – True/False (3 points total)

1. **F** – A consistent formula is true in at least one interpretation, but not necessarily in all.
2. **F** – If A⊨BA⊨B then A→BA→B is tautology, not A→¬B.
3. **F** – We try to derive empty clause from Γ∪{¬C}, not Γ∪{C}Γ∪{C}.
4. **F** – Backward chaining is a resolution method.
5. **F** – If consistent, empty clause **cannot** be derived.
6. **F** – Skolemization is for first-order logic, not propositional.

Each: 1 point (0.25 for T/F + 0.25 for correction if wrong).
Total: 6 points.

Exercise 2 – Propositional Logic (4.5 points total)

Question 2.1: Transform Γ into Horn Clauses 8*0.25 = 2pts

1. **From P→(T∧R):**
   - This splits into two implications: P→T and P→R.
   - **Horn Clauses:** P→T and P→R.
2. **From (S∨T)→P:**
   - This splits into two implications: S→P and T→P.
   - **Horn Clauses:** S→P and T→P.
3. **From T↔U:**
   - This yields two directions: T→U and U→T.
   - **Horn Clauses:** T→U and U→T.
4. **From S:**
   - **Horn Clause:** S.
5. **From ¬U: Nont horn clause**

Final Set of Horn Clauses for Chaining: 7*0.25 = 1.75 pts

- (R1) P→T
- (R2) P→R
- (R3) S→P
- (R4) T→P
- (R5) T→U
- (R6) U→T
- (F1) S (Fact)

Question 2.2: Proving U using Backward Chaining 0.25*3 = .075 pts

Backward chaining starts from the **Goal (U)** and works backward to find facts that support it.

U, R5-----T ?
T, R1-----P ?
P, R3------S true

Exercise 3 – Predicate Logic (12.5 points total)

3.1 – Translation (0.5 *4 = 2 points)

1. Every computer science student likes some module.
2. There is a module such that for all xx, if xx likes it or revises it, then xx passes it.
3. Everyone revises every exam.
4. There is a computer science student.

Grading: 0.5 point each.

3.2 – The conclusion (0.5 point)

C : P(a,ml)

3.3 – Clause form (0.5 +0.5 +0.5) *6 = 9 points)

1. ¬C(x)∨L(x,a)
2. ¬L(x,a)∨P(x,a)
3. ¬R(x,a)∨P(x,a)
4. R(x,y)
5. C(a)

Given F and goal C: P(a,ml)

Add negation: ¬P(a,ml).

CF = {¬C(x)∨L(x,f(x)) , ¬L(x,a)∨P(x,a), ¬R(x,b)∨P(x,b), R(x,y), C(c), ¬P(a,ml)}

3.4 – Refutation (1.5 points)

1. {¬C(x)∨L(x,a) , ¬L(x,a)∨P(x,a)} -----¬C(a) , P(x,a) x=a.
2. C(a) , ¬C(a) , P(x,a) -------P(x,a).
3. P(x,a), ¬P(a,ml).------ ⊥