س1) الإجابة النموذجية امتحان جبر -1- الفصل الأول- $\boxed{\frac{3}{3}}$ ت1)

$$\overline{(p(x)\wedge q(x)) \Leftrightarrow (a(x)\vee b(x))}$$

$$\overline{[(p(x)\wedge q(x)) \Rightarrow (a(x)\vee b(x))]\wedge[(a(x)\vee b(x)) \Rightarrow (p(x)\wedge q(x))]} \equiv$$ 1

$$\equiv \overline{[(p(x)\wedge q(x)) \Rightarrow (a(x)\vee b(x))]}\vee\overline{[(a(x)\vee b(x)) \Rightarrow (p(x)\wedge q(x))]}$$ 1

$$\equiv [(p(x)\wedge q(x))\wedge(\neg a(x)\wedge \neg b(x))]\vee[(a(x)\vee b(x))\wedge(\neg p(x)\vee\neg q(x))]$$ 1

ت2) $\boxed{\frac{3}{3}}$

$(p\wedge q)\vee(\neg p\vee\neg q)$

p	q	¬p	¬q	p∧q	¬p∨¬q	(p∧q)∨(¬p∨¬q)
1	1	0	0	1	0	1
1	0	0	1	0	1	1
0	1	1	0	0	1	1
0	0	1	1	0	1	1

1 1 1

ت3) $\boxed{\frac{4}{4}}$

S3) $A\cap B=\{-1,0,1\}$ 1, $\mathcal{P}(A)=\{\phi,\{-1\},\{0\},\{1\},\{-1,0\},\{0,1\}$
$\{-1,0,1\}\}$ 1

$A\cup B=]-2,2]$ 1 $A-B=\{2\}$ 1

ت4) $\boxed{\frac{5}{5}}$

S4) $f: \mathbb{R}-\{-2\}\longrightarrow \mathbb{R}-\{-1\}$,

if $A=[1,3[$ $f(A)=?$

$$1\leq x<3 \Rightarrow \begin{cases} 3\leq x+2<5 \quad \cdots (1) \\ \wedge \quad 1\leq x<3 \quad (2)\end{cases} \Rightarrow \frac{1}{3}\leq\frac{x}{x+2}<\frac{3}{5}$$ 1

$f^{-1}(B)=\{0,-2,-3\}$.

$f(A)=[\frac{1}{3},\frac{3}{5}[$, 2

هل تقابل إثبات f متباين و شامل.

is f bijection to prove f is injection and f is surjection

1) f is injective متباين

$\forall x_1, x_2\in\mathbb{R}-\{-2\}$, $f(x_1)=f(x_2)\Rightarrow \frac{x_1}{x_1+2}=\frac{x_2}{x_2+2}\Rightarrow$ 1

$x_1x_2+2x_1=x_2x_1+2x_2 \Rightarrow 2x_1=2x_2\Rightarrow x_1=x_2$ then f is injective

2) f is surjective $\forall y\in\mathbb{R}-\{-1\}\ \exists x\in\mathbb{R}-\{-2\}: f(x)=y$

$\frac{x}{x+2}=y \Rightarrow x=xy+2y\Rightarrow x=\frac{2y}{1-y}\in\mathbb{R}-\{-2\}$ as $y\neq -1$ 1 f surjective $\Rightarrow$ f bijective

$\boxed{\frac{5}{5}}$ (5 ن

S 5)

1) $\forall\, x, y \in \mathbb{Z} \quad x R y \Leftrightarrow y \equiv 0\ [3 \bmod]$.

R Reflexive $\quad \forall\, x \in \mathbb{Z},\ x \equiv x\ [\bmod 3]$ R انعكاسية (1)

$\Rightarrow x R x$ انعكاسية

2) Symetric R تناظرية.

$\forall\, x, y \in \mathbb{Z} \quad x R y \Rightarrow y \equiv x\ [3] \Rightarrow \exists\, k / k \in \mathbb{Z},\ y = 3k + x$

$\Rightarrow x = -3k + y = 3(-k) + y$ then $x \equiv y\ [\bmod 3] \Rightarrow y R x$.

R symetric R تناظرية (1)

3) R transitive R متعدية

$\forall\, x, y, z \in \mathbb{Z}$

$\begin{cases} x R y \\ y R z \end{cases} \Rightarrow \begin{matrix} y \equiv x\ [\bmod 3] \\ z \equiv y\ [\bmod 3] \end{matrix} \Rightarrow \left. \begin{matrix} y = 3k_1 + x : & k_1 \in \mathbb{Z} \\ z = 3k_2 + y & k_2 \in \mathbb{Z}. \end{matrix} \right\}$

$\Rightarrow z = 3k_2 + 3k_1 + x = 3(k_1 + k_2) + x$ and $k_1 + k_2 \in \mathbb{Z}$. (1)

$z \equiv x\ [\bmod 3] \Rightarrow x R z$, R transitive,

from (1), (2) and (3) R is equivalent Relation

$\dot{0} = \{ y \in \mathbb{Z},\ 0 R y \} = \{, \ldots, -3, 0, 3, 6, \ldots \}$

$\dot{1} = \{ \ldots, -2, 1, 4, 7, \ldots \}$. (2)

$\dot{2} = \{ \ldots, -1, 2, 5, 8, \ldots \}$.

S6) $G =]-1, 1[$ $\boxed{\frac{5}{5}}$ (3 (6ن

$\forall a, b \in G, \quad a * b = \dfrac{a+b}{1+ab}$.

1) التبديلية

1) $*$ is abelian

$\forall a, b \in G, \quad a*b = b*a$ 1

$a*b = \dfrac{a+b}{1+ab} = \dfrac{b+a}{1+ba} = b*a$.

2/ $*$ is associative تجميعي $*$

$\forall a, b, c \in G, \quad (a*b)*c = a*(b*c)$ 1

$(a*b)*c = \left(\dfrac{a+b}{1+ab}\right)*c = \dfrac{\frac{a+b}{1+ab}+c}{1+c\left(\frac{a+b}{1+ab}\right)} = \dfrac{\frac{a+b+c+abc}{1+ab}}{\frac{1+ab+ca+cb}{1+ab}}$

$= \dfrac{a+b+c+abc}{1+ab+ac+cb}$ ... ①.

$a*(b*c) = a*\left(\dfrac{b+c}{1+bc}\right) = \dfrac{a+\frac{b+c}{1+bc}}{1+a\left(\frac{b+c}{1+bc}\right)} = \dfrac{\frac{a+bca+b+c}{1+bc}}{\frac{1+bc+ab+ac}{1+bc}}$

$= \dfrac{a+bca+b+a}{1+bc+ab+ac}$ -- ② from 1 and 2

$*$ تجميعي.

we have $(a*b)*c = a*(b*c)$

3) عنصر حيادي

3) $\forall a \in G \ \exists e \in G \ / \ a*e = e*a = a$. 1

$a*e = a \Rightarrow \dfrac{a+e}{1+ae} = a \Rightarrow a+e = a+a^2e \Rightarrow (a^2-1)e = 0$.

as $a \in G$ $a^2-1 \neq 0 \Rightarrow e = 0$. عنصر حيادي. 1

4) العناصر النظيرة

4) $\forall a \in G \Rightarrow \exists a' \in G \ /$

$a*a' = a'*a = e \Rightarrow a*a' = 0 \Rightarrow \dfrac{a+a'}{1+aa'} = 0$ as $aa' \neq -1$

$a + a' = 0 \Rightarrow a' = -a$ كل عنصر نظير.

من ①, ②, ③, ④ $(G, *)$ زمرة تبديلية. 1