Hamma-Lakhder university- D'Eloued University
Institute of exact sciences
Exam in Algebra 1. Time 1h30s

date 05/01/2025
Universe year 2024/2025.

تصحيح الامتحان-جبر 1 –اولى اعلام الي -2025/2024.

ج1) عندنا $B=]-1,+\infty[$, $A=(-\infty,2]$, $E=\mathbb{R}$, اذن $CA=A^c=]2,+\infty[$

$A\Delta B=]-\infty,-1]\cup[2,+\infty[$ $\quad A-B=]-\infty,-1]$.

ج2) $\neg\left(\left(\exists x\in N,\left(A(x)\wedge B(x)\right)\Rightarrow\left(\forall x\in N,\left(\neg A(x)\vee B(x)\right)\right)\right)\right)$

$=(\exists x\in N,\left(A(x)\wedge B(x)\right)\wedge(\exists x\in N,(A(x)\wedge\neg B(x)))$

ج3)

p	q	$\neg p$	$\neg q$	p∨ q	$\neg(p\vee q)$	$(\neg p\wedge\neg q)$
1	1	0	0	1	0	0
1	0	0	1	1	0	0
0	1	1	0	1	0	0
0	0	1	1	0	1	1

حسب النتيجة الجملة تكرارية

ج4) $f:\mathbb{R}/\{-1\}\longrightarrow\mathbb{R}$. Defined by $f(x)=\frac{x}{1+x}$ هل f متباين

If $f(x_1)=f(x_2)\Rightarrow\frac{x_1}{1+x_1}=\frac{x_2}{1+x_2}\Rightarrow x_1(x_2+1)=x_2(x_1+1)$

$\Rightarrow x_1x_2+x_1=x_2x_1+x_2\quad\Rightarrow x_1=x_2\quad then\ \ f\ is\ injective$

هل التطبيق غامر

$\forall y\in\mathbb{R}\ ,\quad\exists x\in\mathbb{R}-\{-1\}\quad;f(x)=y$

$f(x)=\frac{x}{1+x}=y\Rightarrow y\,(x+1)=x\quad\Rightarrow x(1-y)=y\Rightarrow x=\frac{y}{1-y}\notin\mathbb{R}-\{-1\}$

that f is not serjective . Now Since f is injective but not surjective is not bijective.

$f(\,]-1\ ,1[\,)=\left]-\infty,\frac{1}{2}\right[$.

ج5) $\forall x\in\mathbb{R}\quad x\,\Re\,y\Leftrightarrow x^2-y^2=x-y.$

1) $\forall x\in\mathbb{R}$, $x=x$ and $x^2=x^2$ then $x^2-x^2=x-x\Rightarrow\ x\Re x$, $\Re$ is reflexive .

2) $\forall x,y\in\mathbb{R}$,we have $x\,\Re\,y\Rightarrow x^2-y^2=x-y\Rightarrow y^2-x^2=y-x\Rightarrow y\,\Re\,x$

then $\Re$ is symmetric .

3)) $\forall x,y,z\in\mathbb{R}$,we have $x\,\Re\,y\ \wedge\ y\,\Re\,z\Rightarrow x\,\Re\,z$
$x\,\Re\,y\ \wedge\ y\,\Re\,z\ \Rightarrow\ (x^2-y^2=x-y\ \wedge\ y^2-z^2=y-z)$ by addition
$\Rightarrow\ x^2-z^2=x-z\Rightarrow x\,\Re z$. then $\Re$ is transitive.

From (1), (2) and (3) $\Re$ is equevalent relation.
The equivalences class of 1 and 2

$\dot{1} = \{x \in \mathbb{R},\ x \Re 1\} \Leftrightarrow x^2 - 1 = x - 1 \Rightarrow x(x-1) = 0 \Rightarrow x = 0 \vee x = 1$.

$$\dot{1} = \{0,1\}.$$

$\dot{2} = \{x \in \mathbb{R},\ x \Re 2\} \Leftrightarrow x^2 - 4 = x - 2 \Rightarrow x = -1 \vee x = 2$

$$\dot{2} = \{-1, 2\}.$$

ج6) نعرف العملية *على الاعداد الحقيقية $\forall x, y \in \mathbb{R},\ x * y = x + y - 3$

1) العملية داخلية $\forall x, y \in \mathbb{R},\ x + y \in \mathbb{R}$

$x + y - 3 \in \mathbb{R}$ *then* $x * y \in \mathbb{R}$.

2) العملية التبديلية اثبات ان $\forall x, y \in \mathbb{R},.\ x * y = y * x$

$x * y = x + y - 3 = y + x - 3 = y * x$

3) العملية التجميعية اثبات ان $\forall x, y, z \in \mathbb{R}, (x * y) * z = x * (y * z)$.

$\forall x, y, z \in \mathbb{R}, (x * y) * z = .(x + y - 3) * z = x + y + z - 6$............(1)

$x * (y * z) = = x * (y + z - 3) = x + y + z - 6$ ....(2) from (1) and (2) we have

$(x * y) * z = x * (y * z)$ then $*$ is associative .

4) العنصر الحيادي $\forall x \in \mathbb{R},\ \exists e \in \mathbb{R} / x * e = e * x = x$ *but* $*$ is abelian then .

$x * e = x \Rightarrow x + e - 3 = x \Rightarrow e = 3$

5) العناصر المتناظرة $\forall x \in \mathbb{R}, \exists \acute{x} \in \mathbb{R} / x * \acute{x} = \acute{x} * x = e.$ as $*$ is abelian then

$x * \acute{x} = 3 \Rightarrow x + \acute{x} - 3 = 3$ *thus* $\acute{x} = 6 - x$

From (1), (2), (3), (4) and (5) that $(\mathbb{R}, *)$ is Abelian group.

ج7) اذا كان G زمرة وتحقق $(ab)^2 = a^2 b^2$ اثبت انها تبديلية

$(ab)^2 = a^2 b^2 \Rightarrow (ab)(ab) = a^2b^2 \Rightarrow a^{-1}(ab)(ab) = a^{-1}a^2b^2 \Rightarrow eb(ab) = e\,ab^2$

$b(ab)b^{-1} = ab^2b^{-1} \Rightarrow ba\,e = abe \Rightarrow ba = ab$. then G be abelian.

s.am. . انتهى