

Answer the Questions

1. The Algorithm (1 + 1 pts):

Algorithm of Dijkstra: Dijkstra's algorithm is a graph-based algorithm used to find the shortest path from a starting node (source) to all other nodes in a graph.

2. The Graph (2 pts):

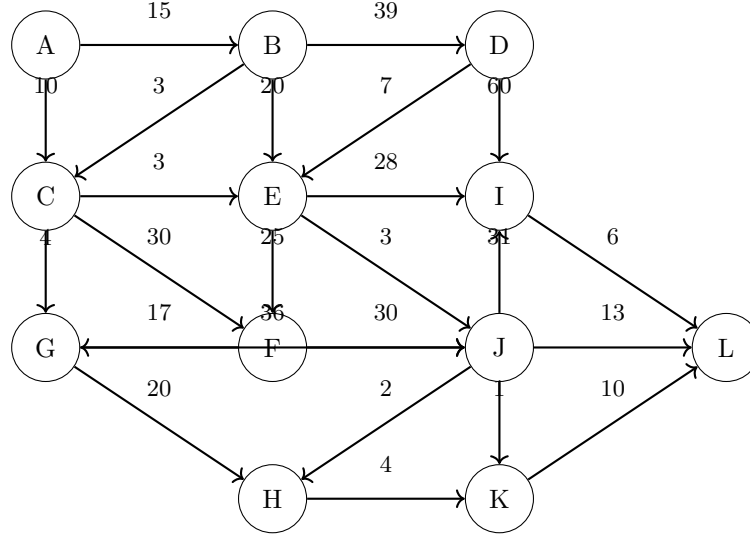


Figure 1: Initial weighted directed graph

3. Graph Matrices (1 + 1 + 1 pts):

Adjacency matrix

$$A = \begin{array}{c|cccccccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ \hline 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$I = \begin{array}{c|cccccccccccccccccccccccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} & e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} & e_{17} & e_{18} & e_{19} & e_{20} & e_{21} & e_{22} & e_{23} & e_{24} \\ \hline 0 & -1 & -1 & 0 \\ 1 & +1 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & +1 & +1 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & +1 & +1 & 0 & 0 & +1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & +1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & -1 & 0 & +1 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & +1 & 0 & 0 & 0 & 0 \\ 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & +1 & 0 & +1 & 0 & 0 & -1 & -1 & -1 & -1 & 0 \\ 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & +1 & 0 & -1 \\ 11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & +1 & +1 \end{array}$$

4. Shortest Paths (3 + 1 pts):

Using the algorithm with source vertex A:

Node	Distance	Shortest Path
A	0	A
B	15	$A \rightarrow B$
C	10	$A \rightarrow C$
D	54	$A \rightarrow B \rightarrow D$
E	13	$A \rightarrow C \rightarrow E$
F	38	$A \rightarrow C \rightarrow E \rightarrow F$
G	14	$A \rightarrow C \rightarrow G$
H	18	$A \rightarrow C \rightarrow E \rightarrow J \rightarrow H$
I	41	$A \rightarrow C \rightarrow E \rightarrow I$
J	16	$A \rightarrow C \rightarrow E \rightarrow J$
K	17	$A \rightarrow C \rightarrow E \rightarrow J \rightarrow K$
L	27	$A \rightarrow C \rightarrow E \rightarrow J \rightarrow K \rightarrow L$

Table 1: Shortest distances and paths from source node A

5. **Task Scheduling Times (6 + 1 pts):** Assuming each edge weight represents the duration of a task

Task	Duration	Le Plus Tôt	Le Plus Tard
$A \rightarrow B$	15	0	0
$A \rightarrow C$	10	0	46
$B \rightarrow C$	3	15	53
$B \rightarrow D$	39	15	15
$B \rightarrow E$	20	15	41
$C \rightarrow E$	3	18	58
$C \rightarrow F$	30	18	56
$C \rightarrow G$	4	18	99
$D \rightarrow E$	7	54	54
$D \rightarrow I$	60	54	110
$E \rightarrow F$	25	61	61
$E \rightarrow I$	28	61	142
$E \rightarrow J$	3	61	136
$F \rightarrow G$	17	86	86
$F \rightarrow J$	30	86	109
$G \rightarrow H$	20	103	142
$G \rightarrow J$	36	103	103
$H \rightarrow K$	4	141	162
$I \rightarrow L$	6	170	170
$J \rightarrow H$	2	139	160
$J \rightarrow I$	31	139	139
$J \rightarrow K$	1	139	165
$J \rightarrow L$	13	139	163
$K \rightarrow L$	10	145	166

Table 2: Tasks with earliest start/finish (Le Plus Tôt) and latest start/finish (Le Plus Tard)

Project Duration : The total duration of the project is **176 days**.

6. **Critical tasks and Path (1 + 1 pts):**

$$A \rightarrow B \rightarrow D \rightarrow E \rightarrow F \rightarrow G \rightarrow J \rightarrow I \rightarrow L$$