

solution Examen Math 2026

Exo 4: 09 pts

$$1) f(x, y) = x^2 + 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x \\ 4y \end{bmatrix} \quad 0,5$$

$$x_{n+1} = x_n - 0,1 \frac{\partial f}{\partial x} \Rightarrow x_{n+1} = x_n - 0,1 \nabla f \quad 0,5$$

$$y_{n+1} = y_n - 0,1 \frac{\partial f}{\partial y}$$

$$(x_0, y_0) = (1, 1) \quad \nabla f(1, 1) = (2, 4)$$

$$1) \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - 0,1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 0,8 \\ 0,6 \end{bmatrix} \quad 1$$

$$2) \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0,8 \\ 0,6 \end{bmatrix} - 0,1 \begin{bmatrix} 1,6 \\ 2,4 \end{bmatrix} = \begin{bmatrix} 0,64 \\ 0,36 \end{bmatrix} \quad 1$$

$$3) \begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0,64 \\ 0,36 \end{bmatrix} - 0,1 \begin{bmatrix} 1,28 \\ 1,44 \end{bmatrix} = \begin{bmatrix} 0,512 \\ 0,216 \end{bmatrix} \quad 1$$

$$Min = f(0,512, 0,216)$$

Max

$$x_{n+1} = x_n + \alpha \nabla f \quad 1.5$$

2) Lagrange Multipliers 0.5

$$\mathcal{L}(x, y, \lambda) = x^2 + 2y^2 - \lambda(x + y - 1)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 2x + \lambda \quad \text{--- (1) } 0.5$$

$$\frac{\partial \mathcal{L}}{\partial y} = 4y + \lambda \quad \text{--- (2) } 0.5$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = x + y - 1 \quad \text{--- (3) } 0.5$$

1.5 { from (1) and (2) $2y = x \Rightarrow$
from (3) $\Rightarrow 2y + y = 1 \Rightarrow y = \frac{1}{3}, x = \frac{2}{3}$
Min $\Rightarrow f(\frac{1}{3}, \frac{2}{3}) = \frac{2}{3}$

Exo 2: 05 pts

$$\begin{cases} x + y + z = 1 \\ 2x + ky + z = 2 \\ 3x + (k+1)y + 2z = 3 \end{cases}$$

1) $AX = b \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & k & 1 \\ 3 & k+1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ 1

2) $\det(A) = 0$ for all k , no unique solution for all k 0.5

3) $\text{rank}(A) = 2$, $\text{rank}(A|b) = 2$ 0.5

- infinitely many solutions for all k 0.5

- no value of k gives no solution 0.5

- no value of k gives unique solution 0.5
+ 0.5

Exo 3: 06 pts

1.5 1 - Rank = number of linearly independent columns

1.5 2 - $\text{Rank}(A) < \text{number of columns} \Rightarrow$
infinite solutions or no solutions

1.5 3 - role of learning rate is to control the step size

1.5 4 - Limitation of GD is sensitive to LR and
can get stuck in local minimum