Exercise 02:

① Yes, this plan admits an optimal transportation plan.

$\Rightarrow \sum a_i = \sum b_j \Rightarrow \sum a_i = 40+42+70 = 24+46+56+26 = 152$ ✓

② Initial basic transportation plan by N.C.M:

	B_1	B_2	B_3	B_4	a_i
A_1	2 (24)	8 (16)	4 ×	6 ×	40
A_2	10 ×	2 (30)	6 (12)	8 ×	42
A_3	4 ×	10 ×	2 (44)	4 (26)	70
b_j	24	46	56	26	152

① * Initial PPlan: $X = \{x_{11}, x_{12}, x_{22}, x_{23}, x_{33}, x_{34}\} = \{24, 16, 30, 12, 44, 26\}$ ✓

* $Z = 24*2 + 16*8 + 30*2 + 12*6 + 44*2 + 26*4 = 500$ ✓

③ Optimal transportation Plan: $c_{ij} = U_i + V_j$ and $D_{ij} = c_{ij} - (U_i + V_j) \geq 0$??

	B_1	B_2	B_3	B_4	a_i	U_i
A_1	2 (24)	8 (16)	4	6 -8	40	0
A_2	10 14	2 (30)	6 (12)	8 0	42	-6
A_3	4 12	10 12	2 (44)	4 (26)	70	-2
b_j	24	46	56	26	152	
V_j	2	8	4	6		

All $D_{ij} \geq 0$ ✓

So optimal plan $= X^* = \{x_{11}, x_{12}, x_{13}, x_{22}, x_{33}, x_{34}\}$

$= \{24, 16, 12, 30, 44, 26\}$ and $Z^* = 500$

④

	B_1	B_2	B_3	B_4	a_i	U_i
A_1	2 (24)	8 (16)	4 (12)	6 0	40	0
A_2	10 14	2 (30)	6	8 8	42	-6
A_3	4 4	10 4	2 (44)	4 (26)	70	-2
b_j	24	46	56	26	152	
V_j	2	8	4	6		

$Z' = 24*2 + 16*8 + 12*4 + 30*2 + 44*2 + 26*4$

$= 500$ ✓

$\Rightarrow$ All $D_{ij} \geq 0$ ✓

So optimal plan

$= X^* = \{x_{11}, x_{12}, x_{13}, x_{22}, x_{33}, x_{34}\}$

$= \{24, 16, 12, 30, 44, 26\}$

and $Z^* = 500$

2nd case: $EC = D_{14}$

$\theta = \min(x_{12}, x_{23}, x_{34}) = (16, 12, 26) = 12$

$LC = x_{23}$

Solution:

Exercise 01: ① Formulation ⇒ ② DLP:

L.P: $\text{Min } Z = 4x_1 + 12x_2$

$$\text{S.t} \begin{cases} 6x_1 + 2x_2 \geq 8 \\ 4x_2 \geq 12 \\ 2x_1 + 4x_2 \geq 24 \\ (x_1, x_2) \geq 0 \end{cases} \quad (2)$$

DLP: $\text{Max } W = 8y_1 + 12y_2 + 24y_3$

$$\begin{cases} 6y_1 + 2y_3 \leq 4 \\ 2y_1 + 4y_2 + 4y_3 \leq 12 \\ y_1 \geq 0 \\ y_2 \geq 0 \\ y_3 \geq 0 \end{cases} \quad (2)$$

③ optimal Solution of DLP: By Simplex

All const $\leq$ + $b \geq 0$ So:

① SF: $\text{Max } W = 8y_1 + 12y_2 + 24y_3 + 0e_1 + 0e_2$

$$\text{S.t} \begin{cases} 6y_1 + 2y_3 + e_1 = 4 \\ 2y_1 + 4y_2 + 4y_3 + e_2 = 12 \\ (y_1, y_2, y_3, e_1, e_2) \geq 0 \end{cases} \quad (2)$$

② B.F.S? (1)

B.V = $(e_1, e_2) = (4, 12)$

N.B.V = $(y_1, y_2, y_3) = (0, 0, 0)$

initial B.F.S: $(y_1, y_2, y_3, e_1, e_2) = (0, 0, 0, 4, 12) \geq 0$

W ⇒ accept B.F.S

③ 1st table: (0.5)

BV	y_1	y_2	y_3	e_1	e_2	b	
e_1	6	0	2	1	0	4	2 Min
e_2	2	4	4	0	1	12	3
-W	8	12	24	0	0	0	

Max

EV = y_3 and LV = e_1

(1)

BV	y_1	y_2	y_3	e_1	e_2	b	
y_3	3	0	1	1/2	0	2	x
e_2	-10	4	0	-2	1	4	1
-W	-64	12	0	-12	0	-48	

EV = y_2 and LV = e_2

(0.5)

BV	y_1	y_2	y_3	e_1	e_2	b
y_3	3	0	1	1/2	0	2
y_2	-5/2	1	0	-1/2	1/4	1
-W	-34	0	0	-6	-3	60

⇒ ALL Coef of W ≤ 0 ; So

$W^* = 60$ and $y_1^* = 0$ (0.5)

$y_2^* = 1$

$y_3^* = 2$

④ The optimal Solution of LP: $Z^* = W^* = 60$ and $x_1^* = -e_1 = 6$

$x_1^* = 6$ (0.5) $x_2^* = -e_2 = 3$

$x_2^* = 3$

⑤ Complementary Slackness Theorem:

$y_i^* ((Ax^*)_i - b_i) \stackrel{?}{=} 0$

$\Rightarrow 0 * (6(6) + 2(3) - 8) = 0$ V

$\Rightarrow 1 * (4(3) - 12) = 0$ V

$\Rightarrow 2 * (2(6) + 4(3) - 24) = 0$ V (0.5)

$\Rightarrow 6(6(0) + 2(2) - 4) = 0$ V

$\Rightarrow 3(2(0) + 4(1) + 4(2) - 12) = 0$ V (0.5)

So, All equations are = 0, So, $x^* = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ and $y^* = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

Are optimal Solution to LP and DLP

Scanned with CamScanner

	B_1	B_2	B_3	B_4	a_i	v_i
A_1	2 (24)	8 (4)	4 0	6 (12)	40	0
A_2	10 14	2 (42)	6 8	8 8	42	-6
A_3	4 4	10 4	2 56	4 (14)	70	2
b_j	24	46	56	26	152	
v_j	2	8	4	6		

⇒ Also: All $\Delta ij \geq 0$

So: optimal Plan 2

$X^* = \{x_{11}, x_{12}, x_{14}, x_{21}, x_{22}, x_{33}, x_{34}\}$

$= \{24, 4, 12, 42, 56, 14\}$

$Z^* = 24*2 + 4*8 + 12*6 + 42*2 + 56*2 +$

$14*4 = 404$

(4) Initial Basic transportation plan by M.C.M:

	B_1	B_2	B_3	B_4	a_i
A_1	2 (24)	8 (4)	4 X	6 (12)	40 ~~164~~
A_2	~~10~~ X	2 (42)	6 X	8 X	~~42~~ 0
A_3	4 X	10 X	2 (56)	4 (14)	~~70~~ 14 0
b_j	~~24~~ 0	~~46~~ 4 0	~~56~~ 0	26 12	152

(1)

$Z = 24*2 + 4*8 + 12*6 + 42*2$

$+ 56*2 + 14*4$

$= 404$ p.f.s

5) Yes, because $Z^*_{NCM} = Z^*_{MCM}$

p.f.s

p.s